CH-5/CONTINUITY& DIFFERENTIABILITY

VERY SHORT ANSWER QUESTIONS (1 MARK)

1. Let f(x)= $\frac{\sqrt{4+x}-2}{x}$, x≠0. For f(x) to be continuous at x=0. Find f(0). Ans: 1/4
2. If y= 1 + x + $x^2/2!$ + $x^3/3!$ + ……..to ∞, then find dy/dx. Ans: y
3. If y = $\cos^{-1}(4x^3-3x)$, then find dy/dx Ans: $-3/\sqrt{1-x^2}$
4. If f(x) = $\begin{cases} \frac{\sin^{-1}x}{x}, x \neq 0 \\ k, \quad x = 0 \end{cases}$ is continuous at x=0, then find the value of k. Ans: 1
5. Find $\frac{d}{dx}[\cos^{-1}(\sqrt{1-x^2})]$ Ans: $1/\sqrt{1-x^2}$
6. Find $\frac{d}{dx}x^x$ Ans: $x^x(1+\log x)$
7. Is Rolle's theorem valid for f(x) = logx on [1,e]? Why? Ans: No
8. Find dy/dx when y= $\sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)$ x>1 Ans: 0
9. Find d^2y/dx^2 when y = logx/x Ans: $(2\log x-3)/x^3$
10. Find $\frac{d}{dx}\left(e^{\sqrt{1+x^2}}\right)$ Ans: $\left(\frac{x}{\sqrt{1+x^2}}\right)e^{\sqrt{1+x^2}}$

SHORT /LONG ANSWER QUESTIONS (4/6 MARK)

11. If f(x) = $\begin{cases} \frac{\sqrt{1+x}-\sqrt{1-x}}{sin-1_x}, x \neq 0 \\ k, x = 0 \end{cases}$ is continuous at x = 0, find k. Ans: 1

12. Find the value of a and b so that the function f(x) = $\begin{cases} x + a\sqrt{2}sinx, 0 \leq x \leq \pi/4 \\ 2xcotx + b, \quad \pi/4 \leq x \leq \pi/2 \\ acos2x - bsinx, \pi/2 < x < \pi \end{cases}$

is continuous for $0 \leq x \leq \pi$. Ans: b = $-\frac{\pi}{12}$, a= -2b

13. Determine the value of 'a' so that f(x) = $\begin{cases} \frac{1-cos4x}{x^2}, if\ x < 0 \\ a, if\ x = 0 \\ \frac{\sqrt{x}}{\sqrt{16-\sqrt{x}}-4}, if\ x > 0 \end{cases}$ is continuous at x = 0

ans a=8

14. Find a and b so that f(x) = $\begin{cases} \frac{\sin(a+1)x+sinx}{x}, x < 0 \\ c, x = 0 \\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{\frac{3}{2}}}, x > 0 \end{cases}$ is continuous at x = 0.

Ans: a = - 3/2, c = ½ , b ≠ 0

15. If $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$ *show that* $d^2y/dx^2 = -b^4/a^2y^3$.

16. If $x^2+y^2 = t-1/t$ and $x^4+y^4 = t^2 + 1/t^2$, then prove that $x^3y\ dy/dx = 1$

17. If $y = \tan^{-1}\left(\frac{5ax}{a^2-6x^2}\right)$, then prove that $\frac{dy}{dx}=\frac{3a}{a^2+9x^2}+\frac{2a}{a^2+4x^2}$

18. Verify L.M.V for $f(x) = \log x$ in [1,2].

19. If $f(x) = \frac{\sqrt{2}\cos x-1}{\cot x-1}$, $x \neq \pi/4$. Find the value of $f(\pi/4)$ so that f becomes continuous at $x = \pi/4$. Ans: ½

20. Let $f(x) = [x] + [-x]$ for $x\neq0$ and $f(0) = \lambda$, if any, is f continuous at x=0.
    Ans: for $\lambda=-1$

21. Show that the function f(x) defined by $f(x) = \begin{cases} \frac{\sin x}{x}+\cos x, & x>0 \\ 2, & x=0 \\ \frac{4(1-\sqrt{1-x})}{x}, & x<0 \end{cases}$ is continuous at x=0.

22. Differentiate the following function w.r.t x: $y= (\sin x)^x + \sin^{-1}\sqrt{x}$.

    Ans: $(\sin x)^x[x\cot x+\log(\sin x)] + \frac{1}{2\sqrt{x-x^2}}$

23. Find dy/dx, if $(x^2+y^2)^2 = xy$. Ans: $\frac{4x(x^2+y^2)-y}{x-4y(x^2+y^2)}$.

24. If $y = \sin^{-1}[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}]$, then find dy/dx. Ans: $\frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x-x^2}}$.

25. Find dy/dx if $y = x^{\cot x} +\frac{2x^2+3}{x^2+x+2}$. Ans: $e^{\cot x}(\cot x/x - \operatorname{cosec}^2 x\log x) + \frac{2x^2+14x+3}{(x^2+x+2)^2}$

26. If $x= \cos t + \log\tan(t/2)$, $y= \sin t$, then find the value of d^2y/dt^2 and d^2y/dx^2 at $t=\pi/4$.

    Ans: $-1/\sqrt{2}$, $2\sqrt{2}$

27. Find the value of k so that the following function is continuous at $x=\pi/2$.

    $f(x)=\begin{cases} \frac{k\cos x}{\pi-2x} & \text{if } x\neq\pi/2 \\ 5, & \text{if } x=\frac{\pi}{2}. \end{cases}$ Ans: k=10

28. Find the derivative of the function $\sqrt{a+\sqrt{a+x}}$ w.r.t x. Ans: $1/(4\sqrt{a+x}\sqrt{a+\sqrt{a+x}})$

29. If $y = \frac{\log(x+\sqrt{x^2+1})}{\sqrt{x^2+1}}$, prove that $(x^2+1)\frac{dy}{dx}+xy=1$.

30. If $y= \log(x+\sqrt{x^2+1})$, prove that $(x^2+1)y_2 + xy_1 = 0$

31. Let $f(x) =\begin{cases} 2+x^3 & \text{for } x\leq1 \\ 3x & \text{for } x>1 \end{cases}$. Verify Lagrange's mean value thm for the function f(x) = [-2,2].

32. Verify Rolle's theorem for the function $f(x) = (x-a)^m(x-b)^n$ in the interval [a,b] where m and n are positive integers.

33. If $\sqrt{1-x^6}+\sqrt{1-y^6}=a(x^3-y^3)$, prove that $\frac{dy}{dx}=\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$

34. If y = $\sqrt{cosx + \sqrt{cosx + \sqrt{cosx + \cdots \propto}}}$ prove that $(2y-1)\frac{dy}{dx}$ + sinx = 0

35. If x = sect – cost and y = $\sec^n t - \cos^n t$, prove that $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$.

36. If y = acos(logx) +bsin(logx). Show that $x^2y_2 + xy_1 + y = 0$.

37. If y =$(\tan^{-1} x)^2$, Prove that $(1+x^2)^2\frac{d^2y}{dx^2}$ +2x((1+x^2)$\frac{dy}{dx}$ = 2.

38. It is given that for the function f(x0 = $x^3 - 6x^2 + px + q$ on[1,3], Rolle's theorem holds with c = 2 + $\frac{1}{\sqrt{3}}$. Find values of p and q. Ans: p = 11, q any number.

39. Examine whether the function f(x) = $x^2 - 6x + 1$ in[1,3] satisfies the hypothesis of Lagranges Mean Value theorem. Hence , find the coordinate of the point at which the tangent is parallel to the chord joiningthe points (1,– 4) and (3, – 8) . Ans: (2 ,– 7)

40. If $f(x) = \begin{cases} x^2 + 3x + a & for\ x \leq 1 \\ bx + 2 & for\ x > 1 \end{cases}$, is differentiable for all real values of x. Find a and b.

Ans : a = 3 , b = 5