

CHAPTER-2/INVERSE TRIGNOMETRIC FUNCTIONS

VERY SHORT ANSWER QUESTIONS (1 MARK)

1. $\cos(\sec^{-1}7 + \operatorname{cosec}^{-1}7)$ is equal to Ans: 0
2. Using principal values, write the value of $2 \cos^{-1}1/2 + 3 \sin^{-1}1/2$ Ans: $7\pi/6$
3. Evaluate: $\tan^{-1}[2 \cos(2 \sin^{-1}1/2)]$ Ans: $\pi/4$
4. Prove that: $\cos(2 \tan^{-1}\frac{1}{7}) = \sin(4 \tan^{-1}\frac{1}{3})$
5. If $\tan^{-1}x + \tan^{-1}y = \pi/4$ where $xy < 1$, find the value of $x+y+xy$ Ans: 1
6. Find the principal value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ Ans: $-\pi/3$
7. Write the value of $\cot[\tan^{-1}\alpha + \cot^{-1}\alpha]$ Ans: 0
8. If $\sin^{-1}(8/17) + \sin^{-1}(3/5) = \sin^{-1}x$, find x. Ans: $77/85$
9. If $\sin[\sin^{-1}(1/5) + \cos^{-1}x] = 1$, find x. Ans: $x=1/5$
10. Solve for x: $\frac{1}{2} \sin^{-1}[2x/(1+x^2)] + \frac{1}{2} \cos^{-1}[(1-x^2)/(1+x^2)] = \pi/3$ Ans: $x=1/\sqrt{3}$
11. Prove that $\cos^2(\tan^{-1}2) + \sin^2(\cot^{-1}3) = 3/10$.
12. Find the principal value of $\tan^{-1}[\sin(\sin^{-1}x + \cos^{-1}x)]$, $x \in [-1, 1]$. Ans: $\pi/4$
13. Prove that $\sec^2(\tan^{-1}2) + \operatorname{cosec}^2(\cot^{-1}3) = 15$.

SHORT ANSWER QUESTIONS (4 MARKS)

14. Show that: $\cot^{-1}(\frac{ab+1}{a-b}) + \cot^{-1}(\frac{bc+1}{b-c}) + \cot^{-1}(\frac{ac+1}{c-a}) = 0$
15. Evaluate: $\operatorname{cosec}[\cos^{-1}(-\frac{12}{13})]$ Ans: $13/5$
16. Solve for x: $3 \sin^{-1}[2x/(1+x^2)] - 4 \cos^{-1}[(1-x^2)/(1+x^2)] + 2 \tan^{-1}[2x/(1-x^2)] = \pi/3$ Ans: $1/\sqrt{3}$
17. Prove that $\sin^{-1}(4/5) + \sin^{-1}(5/13) + \sin^{-1}(16/65) = \pi/2$
18. Prove that $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$
19. Write the following in the simplest form: $\tan^{-1}(\frac{4\sqrt{x}}{1-4x})$ Ans: $2 \tan^{-1}2\sqrt{x}$
20. Prove: $\tan^{-1}\frac{m}{n} - \tan^{-1}\frac{m-n}{m+n} = \frac{\pi}{4}$
21. Find the value of x if $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$ Ans: $\pi/4$
22. Prove that $\tan^{-1}(\frac{\cos x}{1+\sin x}) = \pi/4 - \pi/2$
23. Prove: $3 \tan^{-1}1/4 + \tan^{-1}1/20 + \tan^{-1}1/1985 = \pi/4$
24. Show that $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}] = \sqrt{(x^2+1)}/\sqrt{(x^2+2)}$

25. Find the value of x if $\cot^{-1}\left\{2 \tan\left(\cos^{-1}\frac{5}{13}\right)\right\} + \tan^{-1}\left\{2 \tan\left(\sin^{-1}\frac{5}{13}\right)\right\} \frac{1}{5} = \tan^{-1}4x$ ans. $\frac{27}{278}$

26. If $\cos^{-1}\left(\frac{x}{a}\right) + \cos^{-1}\left(\frac{y}{b}\right) = \alpha$, prove that $\frac{x^2}{a^2} - \frac{2xy}{ab} \cos \alpha + \frac{y^2}{b^2} = \sin^2 \alpha$

27. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, then prove that $x^2 + y^2 + z^2 + 2xyz = 1$

28. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, then prove that $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$

29. Prove that $\tan^{-1}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}} \tan \frac{\theta}{2}\right) = \frac{1}{2} \cos^{-1}\left(\frac{b+a\cos\theta}{a+b\cos\theta}\right)$

30. Prove that $\tan\left(\frac{\pi}{4} + \frac{1}{2} \cos^{-1} \frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2} \cos^{-1} \frac{a}{b}\right) = \frac{2b}{a}$.